TELECOMMUNICATIONS

Electric charge, force and field

Lesson 1 – Class 3TELB

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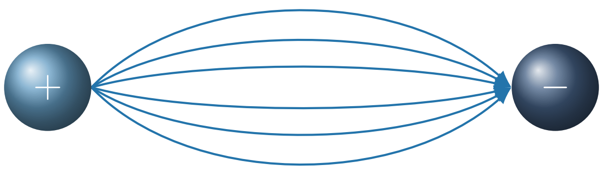
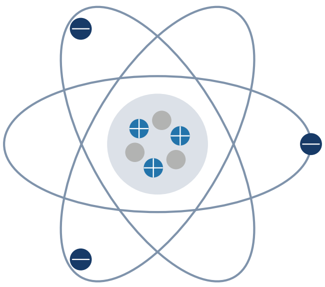


School year 2026/2027

1 Electric charge (*carica elettrica*)

Electric charge (*carica elettrica*) is a property of matter. It is not observed directly; instead, we observe its effects, such as attraction, repulsion, sparks and electric currents. In the International System of Units (SI), charge is measured in **coulombs** (C).

1.1 Particles in the atom (*particelle dell'atomo*)



Proton (*protone*): $+e$

Neutron (*neutrone*): 0

Electron (*elettrone*): $-e$

Electric charge is **quantized** (*quantizzata*): it is a discrete quantity (*grandezza discreta*), so it cannot take just any value. The quantity e is the positive value of the **elementary charge** (*carica elementare*), that is, the smallest charge "step" considered in these phenomena.

Key formula

$$\boxed{e = 1.602 \cdot 10^{-19}\,\mathrm{C}} \qquad \boxed{Q = ne\,, \quad n \in \mathbb{Z}}$$

$$Q_{\text{proton}} = +e \qquad Q_{\text{electron}} = -e = -1.602 \cdot 10^{-19}\,\mathrm{C}$$

Here e is not a charge separate from Q: it is the elementary value used to build every possible value of Q. The total charge of a body can therefore only be

$$\ldots, -3e, -2e, -e, 0, +e, +2e, +3e, \ldots$$

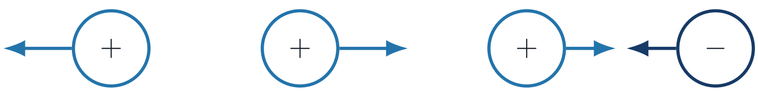
The integer n (*numero intero*) describes the electron imbalance: a negative n means an excess of electrons, while a positive n means a shortage. For example, $Q = -3e = -4.806 \cdot 10^{-19}\,\mathrm{C}$ means that the body has three extra electrons compared with the neutral state. A value such as $Q = 1.5e$ is not allowed.

A body is electrically neutral when it contains equal numbers of protons and electrons. An excess of electrons produces a negative charge; a shortage of electrons produces a positive charge. In the following formulas, Q denotes the total or source charge (*carica sorgente*), while q denotes a generic charge (*carica generica*); if the particle is an electron, then $q = q_e = -e$.

In ordinary electrical phenomena, **electrons** move from one body to another, while protons remain bound inside the nucleus. Charge is neither created nor destroyed: it is transferred. In an isolated system, the algebraic sum of all charges remains constant.

Key concept

Charges with the same sign **repel**; charges with opposite signs **attract**.



repulsion (*repulsione*) attraction (*attrazione*)

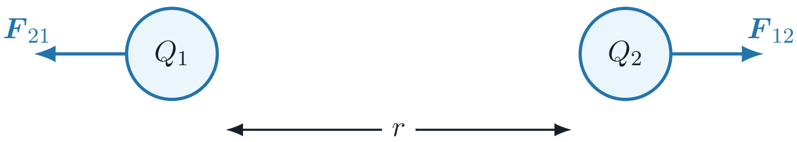
2 Coulomb force (*forza di Coulomb*)

Two **point charges** (*cariche puntiformi*) Q_1 and Q_2, separated by a distance r, exert forces on one another along the straight line joining them.

Key formula

$$\boxed{F = k\,\frac{Q_1 Q_2}{r^2}}$$

$$k = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \qquad k_0 = 8.99 \cdot 10^9\,\mathrm{N\,m^2/C^2} \quad \text{(in vacuum)}$$



example: like charges, repulsive force

How to interpret the formula

- F is directly proportional to the product $Q_1 Q_2$;
- F decreases with the **square** of the distance: if r doubles, F becomes $F/4$;
- the sign of $Q_1 Q_2$ distinguishes the two cases: positive for like charges (repulsive force, *forza repulsiva*) and negative for unlike charges (attractive force, *forza attrattiva*);
- the two forces have equal magnitude and opposite directions: $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$.

The constant k depends on the medium through its **relative permittivity** (*permittività relativa*) ε_r. For the same charges and distance, the force in a material with $\varepsilon_r > 1$ is smaller than in vacuum. Coulomb's law applies to point charges. Outside a uniformly charged sphere, the field is the same as if the entire charge were concentrated at its centre.

Example (*esempio*)

Two charges $Q_1 = 2\,\mu\mathrm{C}$ and $Q_2 = -3\,\mu\mathrm{C}$ are separated by $r = 0.10\,\mathrm{m}$ in vacuum.

$$F = 8.99 \cdot 10^9\,\frac{(2 \cdot 10^{-6})(-3 \cdot 10^{-6})}{(0.10)^2} = -5.39\,\mathrm{N}.$$

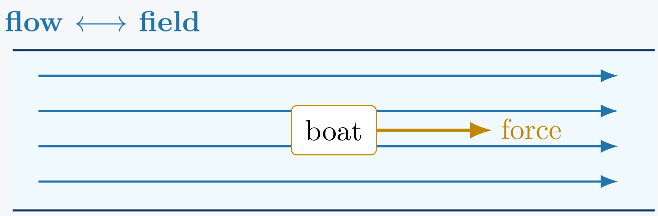
The negative sign indicates an **attractive force**: the charges have opposite signs.

3 The electric field (*campo elettrico*)

One or more charges, called **source charges** (*cariche sorgenti*), modify the surrounding space. At every point in space they establish a vector quantity called the **electric field** (*campo elettrico*).

Analogy: the flow of a river

The water flow exists even when there is no boat in the river. At every point, it has a direction and a strength. The flow is analogous to the **electric field**.
When a boat is placed in the river, the flowing water exerts a force on it. The boat is analogous to a **charge placed in the field** (*carica immersa nel campo*); the force on the boat is analogous to the **electric force** (*forza elettrica*). Without a boat, the water flow still exists, but there is no force on a boat. Likewise, an electric field can exist at a point even when no charge is there.



To determine the field at a point P, we imagine placing a **test charge** (*carica di prova*) q_0 at that point. By convention, q_0 is positive and very small, so it does not appreciably move the source charges or alter the field being measured.

The test charge experiences an electric force $\boldsymbol{F}_0$. The electric field at P is defined as the force per coulomb of positive test charge:

Definition of the electric field – always valid

$$\boxed{\boldsymbol{E} = \frac{\boldsymbol{F}_0}{q_0}} \qquad \boxed{\boldsymbol{F}_0 = q_0\boldsymbol{E}} \qquad [E] = \mathrm{N/C} = \mathrm{V/m}$$

Key concept

The field $\boldsymbol{E}$ depends on the source charges and on the point in space; it does not depend on the test charge. If q_0 doubles, $\boldsymbol{F}_0$ also doubles, but the ratio $\boldsymbol{F}_0/q_0$ remains unchanged. The charge q_0 is only a probe of the field.

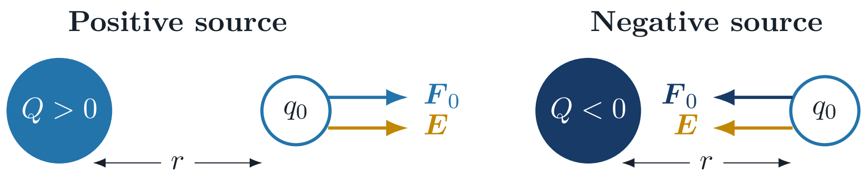
Meaning of each symbol

- Q: the **source charge** (*carica sorgente*); it generates the electric field;
- q_0: a positive, very small **test charge** (*carica di prova*) used to define $\boldsymbol{E}$;
- q: any charge subsequently placed in the field;
- $\boldsymbol{E}$: the field at the point, whether or not a charge is present there;
- $\boldsymbol{F}_0$: the Coulomb force acting on the test charge;
- $\boldsymbol{F} = q\boldsymbol{E}$: the force acting on a generic charge q placed in the field;
- r: the distance between the source charge Q and the point P being considered.

Because q_0 is positive, the direction of $\boldsymbol{E}$ is, by definition, the direction of the force on a positive charge. The force on a negative charge points in the opposite direction to the field.

3.1 Field produced by a point charge (*campo di una carica puntiforme*)

Consider a **point** source charge Q (as in Coulomb's law: its size is negligible compared with the distance r) and a positive test charge q_0 placed at point P, a distance r from Q. The force $\boldsymbol{F}_0$ in the definition $\boldsymbol{E} = \boldsymbol{F}_0/q_0$ is **the same Coulomb force** exerted by Q on q_0.



To derive the field, write the **same force** F_0 in two ways and compare the two expressions:

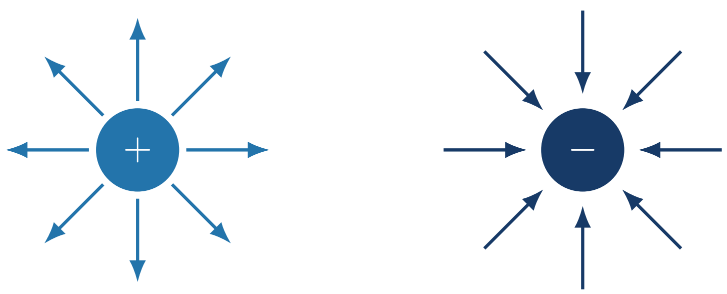
1. from the **definition** of the field, always valid: $F_0 = q_0 E$;
2. from **Coulomb's law**, valid for point charges: $F_0 = k\,\dfrac{Qq_0}{r^2}$;
3. set them equal: $q_0 E = k\,\dfrac{Qq_0}{r^2}$ and divide both sides by q_0.

Field of a point charge – valid only for a point source

$$\boxed{E = k\,\frac{Q}{r^2}}.$$

The test charge q_0 cancels: the field depends on Q and r, not on the probe used to measure it. Because $q_0 > 0$, $\boldsymbol{F}_0$ and $\boldsymbol{E}$ have the same direction.

The sign of Q determines the radial direction of the field: field lines point **away from** a positive charge and **towards** a negative charge. As with the Coulomb force, the sign of the result gives the direction ($E > 0$ outward, $E < 0$ inward), while the field strength is $k\,|Q|/r^2$.



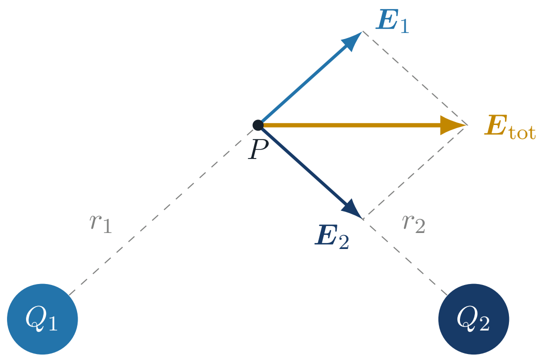
$Q > 0$ $\qquad$ $Q < 0$

3.2 Superposition principle (*principio di sovrapposizione*)

If several source charges $Q_1, Q_2, \ldots, Q_n$ are present, each produces its own field $\boldsymbol{E}_i$ at point P. The total field at P is the **vector sum** of these contributions:

Key formula

$$\boxed{\boldsymbol{E}_{\text{tot}} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \cdots + \boldsymbol{E}_n} \quad \text{or} \quad \boxed{\boldsymbol{E}_{\text{tot}} = \sum_{i=1}^{n} \boldsymbol{E}_i}$$



Calculate $E_i = kQ_i/r_i^2$ separately for each source, including its direction (away from Q_i if positive, towards it if negative, as in the figure); then perform the vector sum. The test charge does not appear in the final result.

Electric field lines (*linee di campo*) are directed curves tangent to $\boldsymbol{E}$ at every point. They never intersect; their density qualitatively represents the field strength.

Which formula should I use?

Formula	What it gives	When it is valid
$F = k\,\dfrac{Q_1Q_2}{r^2}$	force between two charges	two point charges
$\boldsymbol{E} = \dfrac{\boldsymbol{F}_0}{q_0}$	defines the field at a point	always, whatever the source
$\boldsymbol{F} = q\boldsymbol{E}$	force on a charge q placed in a known field	always
$E = k\,\dfrac{Q}{r^2}$	field produced by one charge	a single point source
$\boldsymbol{E}_{\text{tot}} = \sum \boldsymbol{E}_i$	field of several sources	always: **vector** sum of the individual fields